Adjoint and Inverse of matrix

Adjoint of a matrix :-

Let $A = [a_{ij}]_{n\times n}$ be any square matrix and $B = [A_{ij}]$ is a matrix whose elements are the cofactors of the corresponding elements in $|A|$. Then the transpose of B is called adjoint matrix of A.

The adjoint of A is denoted by adj A.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $\therefore B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

The transpose of $B = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$ = adj A.

Ex ① Find the adjoint of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

Soln:-

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{vmatrix}$

The cofactor of $1, 2, 3 = \begin{vmatrix} 5 & 0 \\ 4 & 3 \end{vmatrix}, -\begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix}, \begin{vmatrix} 0 & 5 \\ 2 & 4 \end{vmatrix}$

$= (15-0), -(0-0), (0-10)$

$= 15, 0, -10$

The cofactor of $0, 5, 0 = -\begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix}, +\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$

$= -(6-12), (3-6), -(4-4)$

$= 6, -3, 0$

The cofactor of $2, 4, 3 = +\begin{vmatrix} 2 & 3 \\ 5 & 0 \end{vmatrix}, -\begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix}, +\begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix}$

$= (0-15), -(0-0), (5-0)$

$= -15, 0, 5$

$$\text{adj } A = \begin{bmatrix} 15 & 0 & -10 \\ 6 & -3 & 0 \\ -15 & 0 & 5 \end{bmatrix}' = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}$$

Properties of adjoint

(I) If O is a null matrix then $\text{adj } O = O$

(II) If I is a unit matrix then $\text{adj } I = I$

(III) If A is a square matrix, then $\text{adj } A' = (\text{adj } A)'$

(IV) If A is a square matrix. Then,

$$A(\text{adj } A) = (\text{adj } A) A = |A| I$$

(V) If A is a square matrix then adj A is also symmetric.

Inverse of a matrix :—

Definition :- If A is a non-singular square matrix and the matrix B such that $AB = BA = I$ then B is called inverse of A and denoted by A^{-1}.

$$\therefore \; AA^{-1} = A^{-1}A = I$$

Form rule of adjoint.

$$A(\text{adj} A) = (\text{adj} A) A = |A| I$$

$$\Rightarrow A \cdot \left(\frac{1}{|A|} \text{adj } A\right) = \left(\frac{1}{|A|} \text{adj } A\right) A = I$$

$$\boxed{\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A}, \quad |A| \neq 0 \text{ (which is non-singular)}$$

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Properties: (i) The inverse of matrix is unique

(ii) Reversal law: $(AB)^{-1} = B^{-1}A^{-1}$.

In general, $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$ etc.

(iii) If A is a non-singular square matrix then $(A')^{-1} = (A^{-1})'$

(iv) If the non-singular matrix A is symmetric then A^{-1} is also symmetric.

Ex. (1) Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{bmatrix}$